Flopping between Schrödinger's Cat States

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We show that for an atom in a cavity in the strong coupling regime, flopping between Schrödinger cat states, devised as a superposition of displaced number states, can be accomplished. Beside, the Rabi frequency can be set to zero, so that population trapping or localization can be accomplished. These states could be proved to be useful for quantum computation.

Key words: Quantum Computation; Schrödinger's Cat States; Flopping; Cavity Electrodynamics; Strong Coupling Regime.

Since the work of Cirac and Zoller [1], the relevance of using trapped ions to do quantum computation has been generally acknowledged. Besides, this kind of physical configurations can be used to generate some exotic quantum states, as we are going to see.

In a recent work Moya-Cessa et al. [2] were able to reduce the Hamiltonian of an ion in a Paul trap to the one of the Jaynes-Cummings model [3] without the rotating wave approximation. This has been treated, neglecting the ionic contribution, in [4]. In the following we will keep on naming this model as "Jaynes-Cummings" even if the rotating wave approximation is not applied.

Here, our aim is to find a perturbative solution to the full Jaynes-Cummings Hamiltonian accounting also for the presence of the ion. We want to show how to generate Rabi flopping between a number of displaced states [5]. A superposition of these states can be seen as Schrödinger's cat states, being composed of a superposition of states with a large number of cavity modes, and should be considered in some way as a superposition of macroscopic states. In this way one could be able to maintain coherence for a large number of photons paving the way to quantum computation. The condition to accomplish this effect is to have a strong coupling cavity while keeping the Lamb-Dicke parameter small.

The interaction between a two-level atom and a single mode of radiation is described by the well-known Hamiltonian

$$H = \omega a^\dagger a + \frac{\omega_0}{2}\sigma_3 + g\sigma_1(a + a^\dagger), \tag{1}$$

ω being the frequency of the radiation mode, ω_0 the separation between the levels of the atom, g the coupling between the radiation field and the atom, σ_3 and σ_1 the Pauli matrices and a and $a^\dagger$ the annihilation and creation operators of the radiation field. When the rotating wave approximation is done one gets back the generally used expression for the Jaynes-Cummings model. Indeed, in the weak coupling case, at the resonance, the rotating wave approximation is applied [3]. This means that the off-resonant terms are neglected, being not essential. Indeed, these originate the Bloch-Siegert shift that for optical frequencies is absolutely negligible.

Our aim here is to analyze this model in a strong coupling regime using the dual Dyson series as devised in [6]. In this case we do not apply the rotating wave approximation at this stage. To derive the dual Dyson series for the Jaynes-Cummings model we take as unperturbed Hamiltonian

$$H_0 = \omega a^\dagger a + g\sigma_1(a + a^\dagger) \tag{2}$$

that has as unitary evolution operator

$$U_F(t) = \sum_{n,\lambda} e^{-iE_n t}|[n;\alpha_\lambda]\rangle\langle[n;\alpha_\lambda]||\lambda\rangle\langle\lambda| \tag{3}$$

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with $E_n = n\omega - \frac{g^2}{\omega}$, $\alpha_\lambda = \frac{\lambda g^2}{\omega}$ and

$$|[n;\alpha_\lambda]\rangle = e^{\frac{g}{\omega}\lambda(a-a^\dagger)}|n\rangle, \tag{4}$$

having $a^\dagger a|n\rangle = n|n\rangle$, $\sigma_1|\lambda\rangle = \lambda|\lambda\rangle$ and $\lambda = \pm 1$. It is easily realized that we obtain the unitary evolution operator by displaced number states [5]. Equation (3) is a reformulation of the evolution operator used in [4]. In this way we have reformulated the evolution operator through "dressed states".

Using the expression for $U_F(t)$ we can easily obtain the transformed Hamiltonian as

$$H_F = U_F^\dagger(t)\frac{\omega_0}{2}\sigma_3 U_F(t) \tag{5}$$

that reduces to the form

$$H_F = H_0' + H_1 \tag{6}$$

with

$$H_0' = \frac{\omega_0}{2}\sum_n e^{-\frac{2g^2}{\omega^2}} L_n\left(\frac{4g^2}{\omega^2}\right)\Big[|[n;\alpha_1]\rangle\langle[n;\alpha_{-1}]| \, |1\rangle\langle -1| + |[n;\alpha_{-1}]\rangle\langle[n;\alpha_1]| \, |-1\rangle\langle 1|\Big], \tag{7}$$

L_n being the n-th Laguerre polynomial and

$$H_1 = \frac{\omega_0}{2}\sum_{m,n,m\neq n} e^{-i(n-m)\omega t} \cdot \Big[\langle n|e^{-\frac{2g}{\omega}(a-a^\dagger)}|m\rangle|[n;\alpha_1]\rangle\langle[m;\alpha_{-1}]||1\rangle\langle -1| + \langle n|e^{\frac{2g}{\omega}(a-a^\dagger)}|m\rangle|[n;\alpha_{-1}]\rangle\langle[m;\alpha_1]||-1\rangle\langle 1|\Big]. \tag{8}$$

The Hamiltonian H_0' can be straightforwardly diagonalized by the eigenstates

$$|\psi_n;\sigma\rangle = \frac{1}{\sqrt{2}}\left[\sigma|[n;\alpha_1]\rangle|1\rangle + |[n;\alpha_{-1}]\rangle|-1\rangle\right] \tag{9}$$

that can be seen as a superposition of Schrödinger's cat states with eigenvalues

$$E_{n,\sigma} = \sigma\frac{\omega_0}{2}e^{-\frac{2g^2}{\omega^2}} L_n\left(\frac{4g^2}{\omega^2}\right), \tag{10}$$

where $\sigma = \pm$. The interpretation of this result is straightforward since one sees that for each level of the atom one has an infinite subset of sublevels numbered by the index n. So, the two energy levels can be considered as bands. This is the effect of the coupling with the radiation mode. Then we can write the wave function of the model as

$$|\psi_F(t)\rangle = \sum_{\sigma,n} e^{-iE_{n,\sigma}t} a_{n,\sigma}(t)|\psi_n;\sigma\rangle, \tag{11}$$

and by perturbation theory we can extract the physics of this model in a strong coupling regime through the amplitudes $a_{n,\sigma}(t)$. This gives the equations for the amplitudes

$$i\dot{a}_{m,\sigma'}(t) = \frac{\omega_0}{2}\sum_{n\neq m,\sigma} a_{n,\sigma}(t)e^{-i(E_{n,\sigma}-E_{m,\sigma'})t} \cdot e^{-i(m-n)\omega t}\left[\langle m|e^{-\frac{2g}{\omega}(a-a^\dagger)}|n\rangle\frac{\sigma'}{2} + \langle m|e^{\frac{2g}{\omega}(a-a^\dagger)}|n\rangle\frac{\sigma}{2}\right], \tag{12}$$

so that the quantum theory of resonance of [7] can be applied. Here, one can apply the rotating wave approximation with the resonance condition between two states yielded by

$$E_{n,\sigma} - E_{m,\sigma'} - (n-m)\omega = 0, \tag{13}$$

that gives two different conditions depending on whether σ is different from σ' or not. The first case corresponds to interband transitions and the other to intraband transitions. Both kinds of transition can give rise to Rabi flopping. Then, for interband transitions ($\sigma \neq \sigma'$) one has the Rabi frequency

$$\mathcal{R} = \omega_0\left|\langle n|\sin\left[\frac{2g}{\omega}(a-a^\dagger)\right]|m\rangle\right|, \tag{14}$$

while for intraband transitions one has

$$\mathcal{R}' = \omega_0\left|\langle n|\cos\left[\frac{2g}{\omega}(a-a^\dagger)\right]|m\rangle\right|. \tag{15}$$

We want to study the evolution of the initial state $|0\rangle|g\rangle$. $|0\rangle$ being the vacuum of the field and $|g\rangle$ the ground state of the atom such that $\sigma_3|g\rangle = -|g\rangle$. This initial state can be written as

$$|\psi(0)\rangle = \sum_{n,\sigma} a_{n,\sigma}(0)|\psi_n;\sigma\rangle \tag{16}$$

with

$$a_{n,\sigma}(0) = (\langle g|\langle 0|)|\psi_n;\sigma\rangle \tag{17}$$

$$= \frac{1}{\sqrt{2}}\left[\sigma\langle 0|[n;\alpha_1]\rangle\langle g|1\rangle + \langle 0|[n;\alpha_{-1}]\rangle\langle g|-1\rangle\right].$$

Then

$$a_{n,\sigma}(0) = e^{-\frac{g^2}{2\omega^2}}\left(\frac{g}{\omega}\right)^n \frac{1}{2\sqrt{n!}}[\sigma-(-1)^n]. \tag{18}$$

From (18) we see that for $\sigma = 1$ we have even amplitudes while for $\sigma = -1$ we have odd amplitudes. Then, the initial state can be written as

$$|\psi(0)\rangle = e^{-\frac{g^2}{2\omega^2}}\sum_n\left[\left(\frac{g}{\omega}\right)^{2n}\frac{1}{\sqrt{(2n)!}}|\psi_{2n};1\rangle \right. \tag{19}$$

$$\left. - \left(\frac{g}{\omega}\right)^{2n+1}\frac{1}{\sqrt{(2n+1)!}}|\psi_{2n+1};-1\rangle\right].$$

For interband transitions we suppose e. g. that the initial and final states have the same oddness, so that the system flops with Rabi frequency $\mathcal{R}$ between the amplitudes (e. g. for odd amplitudes $m = 2n + 1$ and $n = 2n + 3$ and $\sigma = -1$ and $\sigma = 1$ respectively)

$$a_{2n+1,-1}(t) = a_{2n+1,-1}(0)\cos\left(\frac{\mathcal{R}}{2}t\right) \tag{20}$$

and

$$a_{2n+3,1}(t) = a_{2n+1,-1}(0)\sin\left(\frac{\mathcal{R}}{2}t\right), \tag{21}$$

that is, the system flops between the states $|\psi_{2n+3};1\rangle$ and $|\psi_{2n+1};-1\rangle$ but population trapping can also be accomplished. This is the final result we wanted to arrive to.

[1] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[2] H. Moya-Cessa, A. Vidiella-Barranco, J. A. Roversi, D. S. Freitas, and S. M. Dutra, Phys. Rev. A **59**, 2518 (1999).
[3] see e. g. M. Scully and M. Zubairy, Quantum Optics, Cambridge University Press, Cambridge 1997.
[4] S. Mancini, H. Moya-Cessa, and P. Tombesi, J. Mod. Optics **47**, 2133 (2000).
[5] F. A. M. de Oliveira, M. S. Kim, P. L. Knight, and V. Bužek, Phys. Rev. A **41**, 2645 (1990).
[6] M. Frasca, Phys. Rev. A **58**, 3439 (1998).
[7] M. Frasca, Phys. Rev. A **58**, 771 (1998).